

# Composition Games for Distributed Systems: the EU Grant games

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## Abstract

In the past, multi-agent systems and networks were designed by some central manufacturer and/or owner. However, increasingly, more modern systems are composed of components, each owned by a different owner. Moreover, such systems are formed rather distributively, by people teaming up to pool their resources together. For example, many Peer to Peer (P2P) networks are composed of nodes belonging to different persons, who would like to gain by cooperation.

In this paper, we consider ways by which people make distributed decisions regarding this composition of such systems, attempting to realize high values. We concentrate on settings in which an agent can increase its utility by connecting to other nodes. However, the agent must also pay a cost that increases with the size of the system. The right balance is achieved by the right size group of agents.

We address this issue using game theory, and refer to games in such settings as European Union grant games (based on the competition for the commission's grants). For such a game, we study its price of anarchy (and also the strong price of anarchy) – the ratio between the average (over the system's components) value of the optimal possible system, and the average value for the system formed in the worst equilibrium. We formulate and analyze three intuitive (and real life) games and show how simple changes in the protocol can improve the price of anarchy drastically. In particular, we identify two important properties for a low price of anarchy: agreement in joining the system, and the possibility of appealing a rejection from a system. We show that the latter property is especially important if there are some pre-existing constraints regarding who may collaborate (or communicate) with whom.

## 1 Introduction

**Background and Motivation.** Often, agents need to team up in groups, when there is some gain from cooperating. For example, in a *peer-to-peer*(P2P) system, the opportunity to share content is the gain from forming the system; in multi-agent systems, agents team up to perform a task and obtain some reward if the task is completed successfully. This motivation to cooperate is a “force” that pushes towards grouping, and has been addressed in numerous papers (some are surveyed below). Here, we are interested in the combination of this force and of a second force that pushes towards breaking large groups into smaller ones. A group that is too large may incur costs that are too high, such as overheads, free loaders, exposure to outside treats (e.g. lawsuits over intellectual properties), etc. This may decrease the value agents get from a large group and motivate them to break it.

We strive to study the above tradeoff, using properties that seem common for such settings. Let us consider the example of FP7, the current, 7th Framework Programme for supporting research in Europe. Its main emphasis is on forming large sets of researchers. The rationale is that a large impact can be realized by a large “network of excellence”. Of course, the commission is not satisfied with size alone, it is interested in the combination of size and quality. How can the commission obtain a “network” satisfying both of these criteria?

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In theory, it seems that the commission could have chosen some set of researchers to its liking, and told those researchers that they now form a research network, so that they cooperate and obtain good research results. It may seem that the commission does possess the tools to choose these researchers wisely. After all, when competing groups of researchers submit grant requests, the commission manages to evaluate which of them is better. This requires being able to evaluate the quality of research and of a researcher. Still, this is not the approach taken by the commission, nor, actually, by other granting agencies. Intuitively, such an approach seems both unpromising and infeasible. First, the values of researchers materializes when they work with researchers with whom they find common grounds and like to cooperate. Second, it is hard for the commission to find a researcher's value unless the researcher exhibits this value first, by submitting a proposal. Third, not all of the researchers are interested in a specific call for proposals. Hence, the commission arranges a competition among sets of researchers that submit grant proposals. In this paper we study the following concrete simple (but realistic) setting that addresses the various properties of such a competition.

First, the winning proposal is the one with the highest “value” of the proposal and of the proposing researchers. We started with a simple model where each researcher has a value, and the value of a set is the average of the values of its members. (This is not far from reality in some cases we observed). Second, the dollar amount of the grant usually grows sub-linearly with the winning set's size (if it grows at all). This may motivate researchers to form a small set. Since the commission is interested in quality larger sets, it adds conditions that require the higher impact of larger groups. To model this, we assume that the commission will not award a grant to a submitting set unless the total of its researcher's values is at least some threshold  $T$ . Third, often, the members of the proposing groups must also show some “synergy”. That is, they are connected by ties of past cooperation, related interests, etc. We model these connections as edges in a graph, where the researchers are the nodes. The “synergy” is modeled by a requirement that the winning group is a connected component.

Researchers that submit a grant as a group also need to decide how to partition the grant money among the group members, if they win. In principle, it may be possible for the agents to bargain on this issue, where “stronger” (in some sense) individuals receive a larger fraction of the total grant. However, in practice, this is rarely the case. Probably due to strong social norms, researchers simply split the money evenly. This happens in many other settings as well. In this paper, we study group composition only under the assumption that the grant is split evenly. While there is no doubt that other ways to split the grant are possible, we defer the study of this issue to future research.

Is this grant game applicable to other cases where people form a set that is a distributed system? This requires further research. Still, let us reconsider the example of a P2P overlay network that is formed by machines, each belonging to a different individual user. The factors mentioned for the above case study seem to be present in such a P2P too. First, these people form the systems out of their own free will. Second, there may not be a grant, but the members do realize some benefit from pooling their resources together. For example, if they exchange music, or movies, they gain by having access to movies held by others. Third, we already noted that a motivation is often present, for these people to form a more exclusive system, rather than to have a very large one.

At this point we need to introduce our game-theoretic framework. While cooperative game theory is usually being employed in the context of coalition formation, this broad theory is mainly concerned with the correct and most efficient way to distribute payoffs among members of a winning coalition, hiding the workings of *how* the winning coalition is formed. In this paper our interests are exactly the opposite. We assume that the payoff distribution is given (and is very specific: the payoff is split evenly). On the other hand, we focus on the question of *what is the composition of the winning group*, and what is its quality. Moreover, the answer to “which group wins” depends not only on the group, but also on the competing groups. For these two reasons, it seems that non-cooperative game theory, together with the notion of the popular *price of anarchy* (POA) ([10, 1]) seems a better fit than cooperative game theory used in most of that

literature. Here, this POA is the ratio between (a) the optimal average value of any eligible set of researchers and (b) the lowest average value of any winner set that can be formed in an equilibrium (here we use both the notions of a Nash equilibrium and of a strong equilibrium).

**Overview of Results.** In this work we introduce and compare three natural methods for deciding the composition of groups, evaluating the “quality” of their equilibrium outcomes relative to the optimal possible outcomes. We also consider the natural constraint of an underlying collaboration network (how does topology affect quality?) and show the importance of an appeals process (in the third method). Intuitively, the games differ in the question of how easy is it for a researcher to join a group if she so desires. The main technical contribution is the analysis of the third game, and the others are given mainly as a basis for comparing the above “ease of joining” versus the obtained average quality.

Broadly speaking, our analysis proceeds as follows. We define the “quality” of a formed set as the average value of its members. We also define the sets that are *eligible* to win. (Recall the constraint of the minimum sum of values, mentioned above, imposed by the granting authority, and the connectivity assumption, imposed by the environment.) The granting agency wishes to award the grant to a group with highest average value among all eligible groups. However, strategic considerations of the researches (who are the players), who aim to obtain the largest possible fraction of the grant, may yield an outcome in which the winning group does not have the highest average quality.

The first (the gold rush game) is a naive composition method that is often used in some legacy systems (e.g. mailing lists): An individual can join a group by simply declaring (unilaterally) her will to do so. This naive method is not usually used by granting agencies and perhaps the reason is that, as we show, the price of anarchy of this method is very high. It is bounded only by the size of the whole society. The analysis of this game is trivial. Besides being a basis for comparison, it also serves as an example.

Actual granting agencies require a stronger condition for researchers to join a group. Usually, they require at least that all group members agree on the composition of the group (the list of participants). We show that if this method is used and the underlying *collaboration network* is a clique, the *strong price of anarchy* improves drastically, to be at most 2. While this improvement is impressive, we show that when the underlying collaboration network is arbitrary (in particular, *not* a clique), the strong price of anarchy of this method can grow up to 3, which means that only a third of the optimal average quality may be realized.

While the source of the high price of anarchy of the first method was the “too easy” joining, the source of the (still rather) high price under the second method is just the opposite – the difficulty of joining. That is, we show examples where some winning set finds it beneficial to disallow the joining of some high value researchers. We introduce a third protocol as a simple alleviation of the previous problems: allowing those rejected high value researchers to “appeal” the rejection. (Interestingly, a granting agency named MAGNET, of the Israeli ministry of industry, uses a similar method for the joining of companies to a consortium). We show that the strong price of anarchy of this third method is lower: at most 2 on arbitrary networks, and even approaches 1 (the optimum) for large sets, if the collaboration network is a complete graph. The analysis of this last method constitutes the main technical contribution of this paper.

**Related Literature.** Probably, the most related paper to ours is [5]. They, too, use non-cooperative game theory. Moreover, in their model, agents have values, and sum of values of a winning coalition must exceed a fixed threshold. However, that model seems to focus on points that do not capture our motivating scenarios. First, we, intentionally, focus on the competition part (that exists even when the payoff depends on the task to be performed, and is independent of the group performing it; we argue that this is rather common- even when the salary is given, one wants to hire the best worker possible). In contrast, there, the values of the players in the winning coalition are also the total payoff of that coalition. Second, we, intentionally, focus on situations where the social norm or physical reality dictates the equal sharing of the grant money; our aim is to characterize the coalitions (and qualities) that will result from the given division the total gain. In contrast, they aim to analyze the bargaining process through which the division of the total gain from the

cooperation will be determined. Finally, our price of anarchy evaluates the values of the winning group. In contrast, the price of democracy they analyze (similar to the price of anarchy) becomes meaningless in our case where there are no costs (costs are an orthogonal parameter introduced there).

[11] studies cooperative games over graphs, where only connected coalitions  $S$  are able to extract their value  $v(S)$ . This is also one of our assumptions. They show that the unique fair way to divide the value of the grand coalition is by the Shapley value. The current paper does not deal with dividing the value. Moreover, here, a group may or may not win, as a function of the the actions of the players in competing groups. This does not seem to be captured well by cooperative game theory.

While most works in classic cooperative game theory are only remotely related to our specific model, a series of papers on the stability of coalition structures, see for example [6], [4], and references therein, is quite relevant to our work. They study a setting where society splits into different coalitions, and characterizes cases where such partitions are stable. (This seems more related than the famous stable marriage problem [8].) Unlike the current paper, they assume that *all* formed coalitions win a prize (otherwise, clearly, no stable coalition structure will emerge). Another difference: these papers do not focus on specific protocols for forming coalitions; their main interest is in characterizing when such stable structures exist. [9] models the creation of coalitions in a game that bears similarities to our initial example game (the gold-rush). However, they do not aim to analyze the quality of the resulting coalition.

Our paper is also related to the literature on network creation games, starting with [7]. These papers study games in which nodes decide how to form links in order to create a connected network, and the price of anarchy is analyzed under various assumptions. In their models, the society becomes *connected*. In contrast, in our paper, the society *splits* to give birth to a *strict subset*, and the question is whether the quality of the formed group is far from optimal because of various strategic issues. The more general literature on the price of anarchy has become quite diverse. A good survey of this literature can be found in the book [12].

## 2 Two Protocols, with Two Opposing Policies

A granting agency wishes to award a prize of  $M$  dollars to some set of researchers that is a subset of a society of  $n$  researchers. Each researcher  $i$  has a value  $v_i$  that represents her overall quality. A subset of researchers is an *eligible subset* if (i) their sum of values is at least some given threshold  $T$ , and (ii) they form a connected component of the underlying Collaboration Network (CN). For simplicity, in most of this section, we assume that the underlying collaboration network is the complete graph, but remove this assumption in subsequent sections, for our main results. The granting agency aims to award the prize to a set of researchers (“consortium”) with maximal average quality among all the eligible consortia. To exclude some trivial cases, we assume throughout the paper that  $v_i < T$  for every researcher  $i$ , i.e. the researcher with the maximal value cannot take the prize on her own. We also assume, without loss of generality, that the sum of all values is larger than  $T$ . While the agency does not know the values of the researchers, we assume that it can verify the values when a set of researchers submits evidence of their value (this is the “grant proposal”). The agency constructs a protocol, by which researchers form candidate consortia, and the best formed consortium wins and receives the prize. We start with two natural protocols that gives some intuition for two possible causes of a high price of anarchy.

**The Gold-Rush Game.** This following protocol, as well as its analysis, are trivial. However, they can serve as a basis to compare our next protocols (as well as a “warm up example”). Each researcher submits a separate proposal, reporting (along with a proof of the researcher’s value) some label, the “consortium name”. The labels are taken from some finite set of labels  $L$ . Researchers who report the same label are understood to belong to the same consortium. The agency awards the prize to an eligible consortium with the maximal average value (if there are ties, the agency uses some arbitrary (possibly randomized) tie-breaking rule known in advance). Each researcher in the winning consortium receives an equal share of the prize.

In terms of game theory, the *strategy* of each researcher  $i$  in this game, is the label  $\ell_i$  she chooses. The *utility*  $u_i(\ell_1, \dots, \ell_n)$  of  $i$  is 0 if “her” consortium loses, and  $\frac{M}{y}$  if her consortium wins, where  $y$  is the size of the winning consortium. A tuple of strategies  $\ell_1, \dots, \ell_n$  is a *Nash equilibrium* if  $u_i(\ell_1, \dots, \ell_n) \geq u_i(\ell_1, \dots, \ell_{i-1}, \ell'_i, \ell_{i+1}, \dots, \ell_n)$  for every  $i = 1, \dots, n$  and every  $\ell'_i \in L$ . In other words, in a Nash equilibrium  $\ell_1, \dots, \ell_n$ , the utility of each researcher  $i$  is maximized by declaring  $\ell_i$ , given that the other researchers declare  $\ell_{-i} = \ell_1, \dots, \ell_{i-1}, \ell_{i+1}, \dots, \ell_n$ . It has become standard in the algorithmic game theory literature to measure the quality of a game/protocol by its *price of anarchy (POA)* [10]. In our case, this is the optimal (largest possible) average value divided by the average value of the winning consortium in the worst Nash equilibrium. (This reflects a worst-case point of view).

Unfortunately, the price of anarchy of the gold-rush game is very high. To show this, it suffices to study the case of distinct values (i.e. no two values are equal) and a complete CN. To analyze the price of anarchy, the next lemma characterizes all Nash equilibria of this game.

**Lemma 1.** *Assume that values are distinct and the CN is the complete graph. Then, in every Nash equilibrium of the gold-rush game either no winning consortium forms, or all researchers declare the same label, hence all researchers win.*

*Proof.* Let us first consider the case where the players assign labels such that there is no winning consortia i.e. no eligible consortium is formed. Clearly, it is a Nash equilibria if and only if no deviation of a single node can lead to the formation of an eligible consortium. Let  $k$  be the size of the smallest subset of players that can form an eligible consortium. Clearly, any partition of players into consortia such that the maximum size of a consortium is  $k - 2$  does not admit any such deviation, and is, therefore, a Nash equilibrium. However, even if the maximum size of consortia is  $k - 1$ , it may be a Nash equilibrium since there may be no single node deviation that can increase the value of a consortium enough to make it eligible.

Now, consider the case when a winning consortium forms. This happens when there is at least one eligible subset. Any tuple of identical labels is a Nash equilibrium, as no single player can be a winning subset by herself, and thus no player has a profitable deviation that increases her utility. It is left to show that if there is a winning consortium then such equilibria are the only possible equilibria. Fix an arbitrary tuple of distinct values  $v_1, \dots, v_n$ , and assume by contradiction that there exists a Nash equilibrium  $\ell_1, \dots, \ell_n$  in which not all the labels are equal. Let  $W$  be the winning consortium, and let  $Z$  be some subset with a highest average value subset such that  $Z \cap W = \emptyset$ . (Note that  $Z \neq \emptyset$ , though it need not be an eligible consortium). Let  $\text{avg}(W)$  and  $\text{avg}(Z)$  be the average values of players in  $W$  and  $Z$  respectively. Thus, either  $Z$  is eligible and  $\text{avg}(W) \geq \text{avg}(Z)$  (equality is possible), or  $Z$  is not eligible. Let  $i \in Z$  be a player with the highest value among all the players in  $Z$ . Since  $i \in Z$ , she loses and  $u_i(\ell_1, \dots, \ell_n) = 0$ .

Now, consider an alternative strategy  $\ell'_i = W$  for player  $i$  (abusing notation, let  $W$  be also the *label* of the winning subset  $W$ ). If the declarations are  $\ell_1, \dots, \ell_{i-1}, \ell'_i, \ell_{i+1}, \dots, \ell_n$ , the subset  $W \cup \{i\}$  is the winner. First,  $v_i > \text{avg}(Z)$  since values are distinct. Second, for any consortium  $X \neq W$ ,  $\text{avg}(Z) \geq \text{avg}(X)$ , by the choice of  $Z$ . Therefore  $\text{avg}(W \cup \{i\}) > \text{avg}(X) \geq \text{avg}(X \setminus \{i\})$  for any eligible consortium  $X \neq W$ . If  $X$  is not eligible, so is  $X \setminus \{i\}$ . Thus,  $W \cup \{i\}$  is indeed the winner. Hence  $u_i(\ell_1, \dots, \ell_{i-1}, \ell'_i, \ell_{i+1}, \dots, \ell_n) > 0$ , which contradicts the assumption that  $\ell_1, \dots, \ell_n$  is a Nash equilibrium, and concludes the proof  $\square$

This Nash equilibrium immediately implies an unbounded price of anarchy, as average of all the values may be arbitrarily lower than the highest average (of some subset).

For example, consider the case that the highest two values are  $T/2 + \epsilon$  and  $T/2$ , and the sum of all the other values is  $\epsilon$ . Then, the highest average of an eligible subset is  $T/2$ , while the average of all the values is  $T/n$  (plus some negligible value). This yields the following theorem:

**Theorem 1.** *The price of anarchy of the gold-rush game is (arbitrarily close to)  $n/2$ .*

*Proof.* We have already argued above, using an example, that the price of anarchy is at least  $n/2$  (up to a negligible value). It is left to show that the price cannot be larger than that. Let  $V$  be the sum of the two maximal values. Since a winning subset must contain at least two players, its average is at most  $V/2$ . The average of all values, on the other hand, is at least  $V/n$ , and the claim follows.  $\square$

There is another, more conceptual problem with the gold-rush game. In reality, researchers (as well as P2P users!) may know each other, and can coordinate a joint deviation from the presumed equilibrium strategy, e.g. the top-value researchers may coordinate to belong to an exclusive consortium. The notion of a Nash equilibrium does not allow such coordinated deviations, and is therefore conceptually weak for our case. A better notion is a *strong (Nash) equilibrium*, which requires that no subset of the players can jointly deviate and increase each of their utilities [3]. Formally, a tuple of strategies  $\ell_1, \dots, \ell_n$  is a *strong equilibrium* if for any  $\ell'_1, \dots, \ell'_n \in L$  there exists a player  $i$  such that  $\ell'_i \neq \ell_i$  and  $u_i(\ell_1, \dots, \ell_n) \geq u_i(\ell'_1, \dots, \ell'_n)$ .

**Lemma 2.** *If there exists an eligible group which is a strict subset of the society, then there does not exist even a single strong equilibrium in the gold-rush game.*

Since any strong equilibrium is also a Nash equilibrium, to prove this lemma we need only check those Nash equilibria of Lemma 1 when a winning consortium forms. If there is only one eligible consortium possible i.e. the set of all players, the formation of this consortium is clearly a strong equilibrium. However, consider the case when there exists an eligible group that is a strict subset of the society. It can be verified that in this case, there is no strong equilibrium. Essentially, this follows for the following reasons: If there was no winner, players will want to deviate to form a winning consortium; If a winner consortium formed, we know from Lemma 1 that it includes all members of the society, however, the optimal group will want to deviate and form its own separate group to reduce the number of winners and increase the payoff of every winner.

**Consensual consortium composition (CCC).** Intuitively, the bad price of anarchy of the gold-rush game resulted from the fact that it was “too easy” for anybody to join a consortium of her liking. We define the following Consensual Consortium Composition (CCC) game, in a first attempt to fix the problems of the previous naive design.

**Strategies:** Each player submits a “proposal:” her value and a list of the researchers in her consortium.

**Outcome:** An eligible consortium of researchers  $X$  satisfies (1; consistency) each researcher in  $X$  submitted  $X$  as her consortium, (2; threshold)  $\sum_{i \in X} v_i \geq T$ , and (3; connectivity) the consortium is connected in the underlying CN. The winning consortium is a submitting eligible consortium with the maximal average value. If there are several such consortia, the winning one has minimal size (again, breaking ties using an arbitrary rule).

Note that in the CCC game, every consortium member has to approve every other member explicitly. This is usually what we see in reality, and our analysis will give a game theoretic rationale for this choice.

As discussed above, the Nash equilibrium concept is not really appropriate in our context. In fact, for the CCC game, a Nash equilibrium is meaningless. The reader can verify that in this game, any partition of the players into consortia will constitute a Nash equilibrium. We focus on the stronger and more appropriate notion of a strong equilibrium, as defined above. Analogous to price of anarchy concept, the strong price of anarchy (SPOA) [1] of a game in our case is the worst case ratio of the optimal (largest possible) average value over the average value of the winning consortium in the worst strong equilibrium. The following theorem bounds the SPOA for this game.

**Theorem 2.** *Assume the CN is the complete graph. Fix an arbitrary tuple of researcher values, and suppose that a minimal eligible consortium with the highest average value has size  $k$ . Then the strong price of anarchy of the CCC game is (arbitrarily close to)  $1 + \frac{1}{k-1}$ . In particular, the SPOA of the CCC game is at most 2.*

*Proof.* Fix a tuple of values  $v_1, \dots, v_n$ . Let  $OPT$  be a minimal eligible consortium with highest average value, and denote  $|OPT| = k$ . Let  $i \in OPT$  be a player with a lowest value among all players in  $OPT$ . Then,  $\sum_{j \in OPT \setminus \{i\}} v_j < T$ : if all values in  $OPT$  are equal, then this follows from the minimality of  $OPT$ . If not all values in  $OPT$  are equal the inequality follows since otherwise  $OPT \setminus \{i\}$  is an eligible consortium with a higher average value than  $OPT$ . The average value of  $OPT \setminus \{i\}$  is, therefore, at most  $\frac{T}{k-1}$ . Thus, the average value of  $OPT$  is also at most  $\frac{T}{k-1}$ .

Now, let  $W$  be the winner consortium in some strong equilibrium. If  $|W| = l < k$  then there exists an eligible consortium with  $l < k$  researchers. Thus, the  $l$  players with  $l$  highest values form an eligible consortium with average value not smaller than that of  $OPT$ , a contradiction. If  $|W| > k$  then  $W$  cannot be a strong equilibrium. This is because players in  $OPT$  can deviate, form a consortium, win, and increase their utility (since the size of the winning subset strictly decreases). Thus,  $|W| = k$ . Since the sum of values of players in  $W$  is at least  $T$ , the average value of players in  $W$  is at least  $\frac{T}{k}$ .

By the previous conclusions, the ratio of the average value of players in  $OPT$  to the average value of players in  $W$  is at most  $\frac{k}{k-1}$ . This proves that the SPOA of the CCC game is at most  $1 + \frac{1}{k-1}$ .

To prove a matching lower bound, an example (for every given  $k$ ) suffices. For this purpose, consider the following tuple of values, for any small enough  $\epsilon > 0$ . There are  $k$  researchers  $1, \dots, k$ , all with the same value  $\frac{T}{k-1} - \epsilon$ , and researcher  $k+1$  with value  $k\epsilon$ . In this case, the eligible consortium with the highest average value (of  $\frac{T}{k-1} - \epsilon$ ) is  $\{1, \dots, k\}$ . Having researchers  $\{1, \dots, k-1, k+1\}$  form one consortium, excluding researcher  $k$ , is a strong equilibrium – one can verify that no subset can deviate and strictly increase each of their utilities. Thus, the SPOA in this case approaches  $\frac{k}{k-1}$  as  $\epsilon$  approaches 0.  $\square$

When the CN is not a complete graph, this theorem is not necessarily true. For another graph topology, Figure 1 shows an example of a CN and player values such that the SPOA is  $3 - \epsilon$ , where  $\epsilon$  is an arbitrarily small constant. We conjecture that 3 is the correct bound. We remark that for the CCC game a strong equilibria always exists. The proof is essentially the same as that of Lemma 4 (for the MAGNET CCC Game), and is thus omitted here.

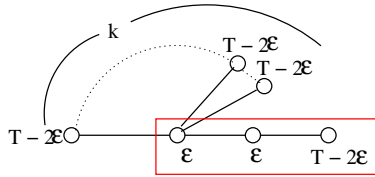


Figure 1: The SPOA for a CCC game on a CN can be arbitrarily close to 3. Here, the nodes are labeled with their values:  $T$  is the threshold,  $\epsilon$  is an arbitrarily small value. The worst equilibria is shown in the box: the other nodes and the central  $\epsilon$  node form the Social Optimum.

The example in the figure also demonstrates another interesting phenomenon: the optimal consortium (with the highest average among all eligible consortia) is not necessarily a strong equilibrium. This follows since the central node will not want to form a consortium with the optimal group, preventing its formation.

### 3 Main Result: MAGNET CCC Game for arbitrary Collaboration Networks

As shown above, the quality of the winning group of the CCC game may be only one third of the optimal possible quality. We now show how to improve the SPOA to 2 for any collaboration network; for the complete graph this price will be close to 1.

For this purpose, we introduce an extension of the CCC game which proceeds over multiple rounds (the name of the game is inspired by a policy of the MAGNET granting agency in the Israel Ministry of Industry; if a consortium of companies wins a grant, but another company can prove to the agency that its inclusion improves the average quality of the consortium, the authority may force the consortium to accept that new company. The rationale is aimed not just at getting good results, but also at following equal access rules). The MAGNET CCC Game is defined as follows:

- In the first round, the CCC game (from the previous section) is performed. Let the winning consortium be  $W_1$ . In each round  $r > 1$  the winning consortium  $W_r$  is an expansion of  $W_{r-1}$ .
- In round  $r > 1$ , each researcher not in  $W_{r-1}$  can “submit an appeal”. This is a proposal, consisting (as in the CCC game) of evidence of her value and a list of researchers in her consortium. The winning consortium  $W_r$  in round  $r$  is the union of  $W_{r-1}$  and all appealing consortia  $X$  that satisfy (1; connectivity)  $X \cup W_r$  is a connected component in CN, (2; consistency), each researcher in  $X$  submitted  $X$  as her consortium, and (3; Improvement)  $average(X \cup W_r) > average(W_{r-1})$ .
- The game ends when  $W_r = W_{r-1}$  (i.e. a fixed point of the improvement process).

In the next two subsections, we analyze the SPOA of this game, first for any arbitrary CN, and then for specific graph structures.

### 3.1 Analysis of SPOA for arbitrary CN

This section shows the main technical result of the paper:

**Theorem 3.** *The MAGNET CCC Game has SPOA of 2 (up to an arbitrarily small  $\epsilon$ ).*

To prove this theorem we first identify some properties that any winning consortium in the MAGNET CCC Game must have. Throughout, we denote by SOW (Social Optimum Winner) a minimal eligible consortium among all eligible consortia with maximal average value.

**Lemma 3.** *Fix an arbitrary instance of the MAGNET CCC Game, and let  $Z$  be the winning consortium in some strong equilibrium outcome of the MAGNET CCC Game. Then the following properties hold:*

1.  $Z \cap SOW \neq \emptyset$
2.  $|Z| \leq |SOW|$
3.  $avg(Z) \geq avg(SOW \setminus Z)$

*Proof.* 1.  $Z \cap SOW \neq \emptyset$ :  $SOW$  is the consortium with the highest average. If  $Z$  is not an SOW, the only way  $Z$  can win is if it can prevent the formation of  $SOW$ , and this can only happen if  $Z$  has some member(s) of  $SOW$  i.e.  $Z \cap SOW \neq \emptyset$ .

2.  $|Z| \leq |SOW|$ : We showed (part 1) that  $Z \cap SOW \neq \emptyset$ . Assume  $|Z| > |SOW|$ . Now, the players in  $Z \cap SOW$  could have improved their utility by forming the smaller consortium  $SOW$  in the first round, which is a sure winner (having the highest average). Thus,  $|Z| \leq |SOW|$ .
3.  $avg(Z) \geq avg(SOW \setminus Z)$ : If  $avg(Z) < avg(SOW \setminus Z)$ , then  $Z$  cannot be a strong equilibrium outcome since players in  $SOW \setminus Z$  have a deviation that makes them winners – appeal together after the currently last round. Since both  $SOW$  and  $Z$  are connected then so is  $SOW \cup Z$ , and since  $avg(Z) < avg(SOW \setminus Z)$ ,  $SOW \setminus Z$  can be added as winners.

□



**Lemma 4.** *The MAGNET CCC Game always has a Strong Equilibrium (S.E.)*

*Proof.* The game always has a *SOW*. This may be a S.E. itself. If it is not, this implies that there is another possible winning consortium  $Z$  having non-empty intersection with *SOW* and of size smaller than *SOW* (else there is no incentive to deviate). Suppose that this is not a S.E. too. Again, this implies the existence of a consortium  $Z'$  having non-empty intersection with  $Z$  and of size smaller than  $Z$ , and so on. Since this size reduction can happen only a finite number of times (the minimum size of an eligible group is 2), we are sure to have a consortium  $Z'$  that allows no profitable deviation, and hence, the game has a strong equilibrium.  $\square$

**Theorem 4.** *The SPOA of MAGNET CCC Game is at most 2.*

*Proof.* As above, let *SOW* be the optimal consortium and  $Z$  be the winning consortium in the worst strong equilibrium outcome of the game. Denote  $|SOW| = k$ . By definition,

$$SPOA = \frac{avg(SOW)}{avg(Z)} = \frac{\frac{sum(SOW \cap Z)}{k} + \frac{sum(SOW \setminus Z)}{k}}{avg(Z)} \quad (1)$$

We prove the lemma by showing that (1)  $sum(SOW \cap Z)/k \leq avg(Z)$  and (2)  $sum(SOW \setminus Z)/k \leq avg(Z)$ .

1.  $sum(SOW \cap Z)/k < avg(Z)$  : obviously  $sum(SOW \cap Z) \leq sum(Z)$ . By Lemma 3, we also have  $|Z| \leq k$ . Thus,

$$\frac{sum(SOW \cap Z)}{k} \leq \frac{sum(Z)}{k} \leq \frac{sum(Z)}{|Z|} = avg(Z)$$

2.  $sum(SOW \setminus Z)/k \leq avg(Z)$  : By Lemma 3,  $avg(Z) \geq avg(SOW \setminus Z)$ . Thus,

$$\begin{aligned} \frac{sum(SOW \setminus Z)}{k} &\leq \frac{sum(SOW \setminus Z)}{|SOW \setminus Z|} \\ &= avg(SOW \setminus Z) \leq avg(Z) \end{aligned}$$

Plugging into equation 1, we get:

$$\begin{aligned} SPOA &= \frac{sum(SOW \cap Z)/k + sum(SOW \setminus Z)/k}{avg(Z)} \\ &\leq \frac{avg(Z) + avg(Z)}{avg(Z)} = 2 \end{aligned}$$

$\square$

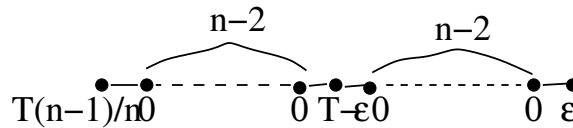


Figure 2: For the MAGNET CCC Game a Collaboration Network with a price of anarchy arbitrarily close to 2.

Fig. 2 shows an example with  $SPOA = 2 - \epsilon$ , where  $\epsilon$  is an arbitrarily small constant. The *SOW* consists of the two players with values  $\frac{T(n-1)}{n}$  and  $T - \epsilon$ , coupled with their intermediary players that have zero values. The worst strong equilibrium here is the consortium containing the two players with values  $\epsilon$  and  $T - \epsilon$ , coupled with their intermediary players that have zero values. This gives the  $SPOA = \frac{2n-1}{n} - \frac{\epsilon}{T}$ , which can be made arbitrarily close to 2. This concludes the proof of theorem 3.

### 3.2 Dependency on graph parameters

The previous analysis showed a SPOA of 2 for arbitrary graph structures. In this section, we show how the SPOA may depend on the *structure* of the graph. In particular, we consider two extreme special cases: the complete graph on one hand, and the network graph on the other hand. These two cases result from differences in the diameter and the connectivity.

**The Complete Graph.** This case was studied previously (in the section on CCC games), showing a SPOA of  $1 + \frac{1}{k-1}$  for the *one round* CCC game, where  $k = |SOW|$ . Here, we show that the multiple round version improves this bound to be  $1 + \frac{1}{k}$ . While the improvement is small for large *SOW*'s, for small *SOW*'s it is quite significant, e.g. the SPOA reduces from 2 (with one round) to 1.5 (with multiple rounds).

**Theorem 5.** *The SPOA of the MAGNET CCC Game over a complete CG is exactly  $1 + \frac{1}{k}$ , where  $k = |SOW|$ .*

We prove the theorem in several steps, starting with two easy observations. Throughout, without loss of generality, we assume that  $v_1 \geq v_2 \geq \dots \geq v_n$ . No generality is lost, since in the complete graph, every player can collaborate with every other player, so we can rename the players as we wish. The first observation is extracted from the proof of Theorem 2.

**Observation 1.** *For the case of the complete graph, every SOW consortium has size  $k$ , where  $k$  is such that  $\sum_{i=1}^{k-1} v_i < T$  and  $\sum_{i=1}^k v_i \geq T$ .*

The second observation is an immediate consequence of the second property of Lemma 3, coupled with the previous observation.

**Observation 2.** *For the case of the complete graph, the size of any winner consortium in a strong equilibrium outcome must be equal to the size of the SOW consortium.*

We now prove Theorem 5 by the following two lemmas.

**Lemma 5.** *The SPOA of the MAGNET CCC Game over a complete CN is at most  $1 + \frac{1}{k}$ , where  $k = |SOW|$ .*

*Proof.* Let  $W$  be the non-optimal winner subset in a worst strong equilibrium. By the above observation,  $W$  is of size  $k$ . Thus, there exists a researcher  $i \in SOW \setminus W$ . Now,  $v_i \leq \text{sum}(W)/k$ , else  $i$  can be added to  $W$  by the improvement process, contradicting the definition of  $i$  as a researcher not in  $W$ . Thus,  $v_k \leq v_i \leq \text{sum}(W)/k$ . Since  $W$  is a winning consortium,  $\text{sum}(W) \geq T > \sum_{j=1}^{k-1} v_j$ . We get,

$$\begin{aligned} \text{SPOA} &= \frac{\text{sum}(SOW)/k}{\text{sum}(W)/k} = \frac{\sum_{j=1}^k v_j}{\text{sum}(W)} \\ &= \frac{\sum_{j=1}^{k-1} v_j}{\text{sum}(W)} + \frac{v_k}{\text{sum}(W)} \leq 1 + \frac{1}{k} \end{aligned}$$

□

**Lemma 6.** *There exists an instance of the MAGNET CCC Game over a complete CN for which the strong price of anarchy is arbitrarily close to  $1 + \frac{1}{k}$ , where  $k = |SOW|$ .*

*Proof.* Fix  $k$ , and consider the following tuple of values, for any  $\epsilon > 0$ : There are  $k - 2$  researchers  $1, \dots, k - 2$  with the same value  $\frac{T}{k-1}$ . Researchers  $k - 1, k, k + 1$  have values  $\frac{T}{k-1} - \epsilon, \frac{T}{k}$ , and  $\epsilon$  respectively. In this case, the eligible consortium with the highest average is  $1, \dots, k$  having average  $\frac{T+T/k-\epsilon}{k}$ . The worst strong equilibrium has the winner as the consortium  $1, \dots, k - 1, k + 1$ . This has average  $\frac{T}{k}$ , hence the strong price of anarchy approaches  $1 + \frac{1}{k}$  as  $\epsilon$  approaches 0. □

**The Line Network.** This is the other extreme case. Here, the price of anarchy *grows* and approaches 2 as the size of the SOW increases (Theorem 6),. This is in contrast to the case of the complete graph where (as shown above) the SPOA *shrinks* and approaches 1 as the size of the SOW increases. Intuitively, it seems that this 100% increase (from the optimum) in the case of a line results from the growth of the diameter when  $k$  grows. On the other hand, the disappearance of the price of anarchy (the convergence to 1) in the case of a complete graph seems to be coming from the increase in the connectivity.

**Theorem 6.** *In the MAGNET CCC Game over a line CN, the SPOA is (arbitrarily close to)  $1 + \frac{k-1}{k}$ , where  $k = |SOW|$ .*

*Proof.* Let  $W$  be the winner in a worst strong equilibrium, and let  $k' = |SOW \setminus W|$ . By Lemma 3, we know that  $k' \leq k - 1$  (since  $SOW \cap W$  is not empty). By the same lemma,  $avg(SOW \setminus W) \leq avg(W)$ , hence  $sum(SOW \setminus W) \leq k' \cdot avg(W)$ . Once again, by the same lemma,  $|W| \leq k$ . In addition,  $k \cdot avg(W) \geq sum(W) > sum(SOW \cap W)$ . All these inequalities imply:

$$\begin{aligned}
SPOA &= \frac{avg(SOW)}{avg(W)} \\
&= \frac{\frac{sum(SOW \setminus W)}{k} + \frac{sum(SOW \cap W)}{k}}{avg(W)} \\
&\leq \frac{\frac{k' \cdot avg(W)}{k} + \frac{sum(SOW \cap W)}{k}}{avg(W)} \\
&= \frac{k'}{k} + \frac{sum(SOW \cap W)}{k \cdot avg(W)} \leq \frac{k-1}{k} + 1.
\end{aligned}$$

The other direction is shown via the example in Figure 2, using  $|SOW| = n$ . Specifically, the proof of Lemma 4 shows that the SPOA there is  $\frac{2n-1}{n} - \frac{\epsilon}{T} = 1 + \frac{k-1}{k} - \frac{\epsilon}{T}$  for any arbitrarily small  $\epsilon > 0$ .  $\square$

Figure 3 shows the different growth curves of SPOA for the complete and line graph as a function of the size of optimal set.

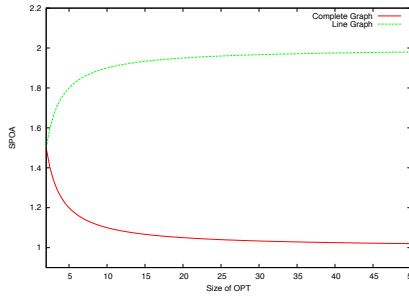


Figure 3: *Strong Price of Anarchy (SPOA) as a function of consortium size  $k$* : For the complete graph, SPOA decreases with  $k$ , but for the line graph, it increases.

### 3.3 Optimality of the results

Let us compare the SPOA of MAGNET CCC Game to other protocols where the designer's aim is to allocate a grant among consortia (that induce a connected component on the  $CN$ , having a threshold of  $T$  with no

single individual meeting the threshold) and where the grant is equally divided among the members of the winning consortia. Let us call such protocols *grant protocols*. We consider protocols which follow the following constraints:

- C1: The designer is unaware of the of the individual values of players and of the topology of the communication network (CN). However, the designer may invite grant proposals (consortia apply for the grant and each player participates in exactly one consortium) and, in that case, can assess the average value of each of the consortium. Notice that this is one of the central assumptions of our model: the only action that the designer can take to learn about the collaboration network and participant values is to invite proposals from participants.
- C2: Once the designer chooses a player as part of a winning consortium, it cannot drop that player from the winning consortium. This requirement can be thought of as a normative property: it would not seem justified of an agency to remove announced winners. Moreover, if we were to allow removals on top of additions (by our appeal process), we can easily construct a protocol with  $SPOA = 1$  by figuring out individual node values and then forcing the formation of the social optimum.

The following theorem shows that *MAGNET CCC Game* is optimal wrt SPOA for all protocols satisfying the above conditions.

**Theorem 7.** *Any grant protocol that satisfies properties C1 and C2 has SPOA at least  $1 + \frac{k-1}{k} - \epsilon$ , where  $k = |SOW|$ , and  $\epsilon > 0$  is an arbitrary small constant.*

*Proof.* Consider an instance of the game on the Collaboration Network as in Figure 2. We know that any protocol that makes in the first step the same choice as the MAGNET CCC Game has a SPOA of at least  $1 + \frac{k-1}{k} - \epsilon$  on this instance when it chooses the winning consortium as the central player (with value  $T - \epsilon$ ) and the nodes on its right. Consider a protocol that makes a different choice: it has to include the leftmost node (with value  $T \frac{n-1}{n}$ ). However, since the designer has no knowledge of the topology of the network, it cannot distinguish the above CN from another CN where there are an arbitrary number of players (with value zero) between the leftmost player and the next nonzero player (i.e the player with value  $T - \epsilon$ ). This choice has arbitrarily low average and thus, arbitrarily high SPOA. Thus, any protocol that follows the above properties will have SPOA at least as high as the MAGNET CCC Game.  $\square$

### 3.4 Additional Issues

This section briefly discusses some additional issues which we do not discuss in detail in this paper, but leave them for the full version.

#### 3.4.1 SPOA of subgame perfect equilibria

Here, we show that for the MAGNET CCC Game, even if we consider Strong Subgame Perfect Equilibria (SSPE) instead of Strong Equilibria, we still get the same price of anarchy. First, we show a simple lemma.

**Lemma 7.** *In the MAGNET CCC Game, Consider a Strong Equilibrium. Let  $W$  be the winner consortium in this equilibrium. Then, it follows that we also have a Strong Subgame Perfect Equilibrium where  $W$  is the winner consortium.*

*Proof.* Consider a strong equilibrium of the MAGNET CCC Game where  $W$  is the eventual winner consortium. Now, consider another instance in which  $W$  is formed in the first round itself (this is clearly possible in MAGNET CCC Game). It is easy to see that this itself is a strong equilibrium as well as a subgame perfect strong equilibrium.  $\square$

**Theorem 8.** *For SSPE, the MAGNET CCC Game has a SPOA of 2 (upto an arbitrary small  $\epsilon$ )*

*Proof.* Every SSPE is also a SE, therefore the SPOA of SSPE cannot exceed that of SE. From Lemma 7 and Theorem 3, the result follows.  $\square$

### 3.4.2 Strong Price of Stability

Another useful concept analogous to price of anarchy is the price of stability (POS) [2]. Analogous to SPOA, one can define Strong Price of Stability (SPOS) as the ratio of the optimum to a best strong equilibrium. In our game, this is the ratio of the optimal (largest possible) average value to the average value of the winning consortium in the best strong equilibrium. We do not analyse SPOS in detail here but do show an example (Figure 4) that shows that in MAGNET CCC Game, the SPOS can be greater than 1.

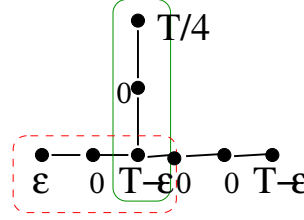


Figure 4: The Strong Price of Stability for the MAGNET CCC Game can be greater than 1. Figure shows the best Strong Equilibria (Green, solid lines) that yields a SPOS of 6/5. For comparison, the worst Strong Equilibria is shown in Red, dashed lines showing a SPOA of 3/2.

## 4 MAGNET 3-Non-Zeros game

To analyze the MAGNET game further and to try to elucidate the relationship with a graph parameter, we introduce a useful restriction of the MAGNET game and prove some of its properties.

The MAGNET- 3 Non Zeros game (3NZ, for short) is the MAGNET CCC game restricted to the condition that only 3 players (nodes) have non-zero positive values, the rest being zero. This is an artificial restriction of the game, meant to elucidate the worst case, in some way, over various CN topologies. Notice that at least 3 non-zero value players are needed to obtain a non-trivial price of anarchy.

Let the three players be  $p_x, p_m, p_\ell$  i.e the set of NonZero players  $NZ = \{p_x, p_m, p_\ell\}$ . Let the values of these nodes i.e.  $\{p_x, p_m, p_\ell\}$  be  $\{x, m, \ell\}$  respectively. Consider the cases where there can be winners other than the Social Optimum Winner(SOW) since these are the only interesting case for us (absence of such a case implies that the POA is 1, which happens in only limited cases). We are interested in finding the highest SPOA for a particular topology over all possible arrangement of players in that topology. Let  $p_m \in SO \cap W$ ,  $SO \cap NZ = \{p_x, p_m\}$ , and  $W \cap NZ = \{p_m, p_\ell\}$ . It's easy to see that without loss of generality, this is the only arrangement possible since  $x, m$ , and  $\ell$  are all less than threshold  $T$ .

First, we prove a small lemma:

**Lemma 8.**  $x \leq \frac{dist(p_x, W)}{Size(W)} Sum(W)$ , where  $W$  is a winner set,  $dist(p_x, W)$  is the shortest distance between  $p_x$  and  $W$  on the CG.

*Proof.* Consider the elements on the shortest path between  $p_x$  and  $W$  including  $p_x$  but no member of  $W$ . The average of this set will be less than the average of  $W$  i.e.

$$\frac{x}{dist(p_x, W)} \leq \frac{Sum(W)}{Size(W)}$$

Therefore,

$$x \leq \frac{\text{dist}(p_x, W)}{\text{Size}(W)} \text{Sum}(W)$$

□

Now, we prove the main theorem:

**Theorem 9.** For the 3NZ game over a CG  $G$ ,  $SPOA \leq \frac{\text{dist}(p_x, W) + \text{size}(W)}{k}$ , where  $\text{Size}(SOW) = k$ ,  $W$  is a winner set.

*Proof.* Putting the expression for  $SPOA$ , we get:

$$\begin{aligned} SPOA &= \frac{\text{avg}(SOW)}{\text{avg}(W)} = \frac{(x + m)/k}{(m + \ell)/\text{size}(W)} \\ &\leq \left( \frac{\text{dist}(p_x, W) \text{Sum}(W)}{\text{Size}(W)} + m \right) \frac{\text{size}(W)}{(m + \ell)k} \\ &< \left( \frac{\text{dist}(p_x, W)}{\text{size}(W)} + 1 \right) \frac{\text{size}(W)}{k} \\ &= \frac{\text{dist}(p_x, W) + \text{size}(W)}{k} \end{aligned}$$

where the second line follows from Lemma 8 and the third line follows from the fact that  $m + \ell = \text{Sum}(W)$ . □

#### 4.1 Cartwheel Graph

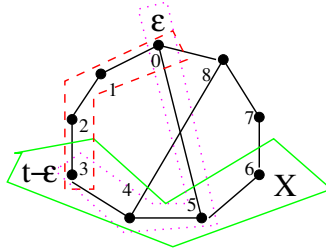


Figure 5: A cartwheel graph with  $n = 9$  with size of the optimal set  $k = 4$ . The optimal set is the solid bounded (green in color) subset (nodes 3-6). If the winner set  $W$  is the subset of nodes (0-3),  $SPOA$  is bounded by 1.5, but for  $W$  as (0, 3 - 5), the  $SPOA$  is bounded by 1.25.

As was observed earlier(Figure 3), the strong price of anarchy is proportional to the size  $k$  of the social optimal set for a line graph and inversely proportional for the complete graph. We suspect this is somehow connected to the diameter and connectivity of the graphs. It will be interesting to look at intermediate topologies which shed more light; In particular, here, we show a graph topology (introduced below), with almost the same diameter and only slightly more connected than the cycle graph. We call this topology the *cartwheel graph* (Figures 5 and 6). The cartwheel graph is a cycle with two intersecting chords such that both the endpoints of both the chords are neighbors. More formally:

**CartWheel Graph:** A graph consisting of a cycle graph  $C_n$  over  $n$  nodes 0 to  $n - 1$ , and chords connecting nodes 0 and  $\lceil \frac{n}{2} \rceil$ , and nodes  $n - 1$  and  $\lceil \frac{n}{2} - 1 \rceil$ .

Informally, it is interesting to note that for the Cartwheel graph over 9 nodes(using an example), in the 3NZ game, we achieve a higher SPOA for an arrangement with the Size of Optimum,  $k$  being 3(Figure 6),

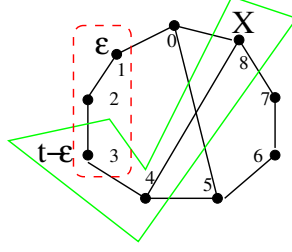


Figure 6: A cartwheel graph with  $n = 9$  with size of the optimal set  $k = 3$ . The optimal set is the solid bounded (green in color) subset (nodes 3,4,8 ). The winner set  $W$  is the subset (nodes 1-3),  $SPOA$  is bounded by 1.66

rather than for  $k$  being 4(Figure 5). Refer to the graph in figure 5. This is a cartwheel graph over 9 nodes with the three non-zero nodes at node 1 (with value  $\epsilon$ ), node 3 (value  $t - \epsilon$ ), and node 6 (with value  $X$ , which is a variable fixed as explained later), and the threshold as  $t$ . Call the subset of nodes  $0 - 3$  as  $W_1$ , nodes 3, 4, 5, 0 as  $W_2$  and nodes 3 - 6 as  $P$ . The value of node 6 i.e.  $X$  determines the possible winners and optimum set e.g. if  $X > t/2$ ,  $P$  is the only possible winner and therefore,  $SPOA = 1$ . Consider  $t/4 < X \leq t/2$ ;  $P$  is the socially optimum set (therefore,  $k = 4$ ),  $W_1$  is now a possible winner. From theorem 9 and the facts that  $dist(node6, W_1) = 2$ , we get  $SPOA \leq 1.5$ . If  $\epsilon < X \leq t/4$ , even  $W_2$  is a possible winner, but the  $SPOA$  drops to 1.25. For  $k = 4$ , it seems that the maximum  $SPOA$  is 1.5. However, consider Figure 6. Again applying theorem 9, with the winner set as nodes 1,2,3 and the social optimum as nodes 3,4,8(therefore,  $k = 3$ ), we get the  $SPOA$  as 1.66.

## 5 Conclusions and Future Work

This paper looks at the process of agents teaming up to construct distributed systems. Our setting addresses a specific scenario where one driving force/ incentive limits the size of the consortium, but another increases it. We made some simple assumptions. We assumed that the value of a researcher is independent of the members of its consortium. We also assumed that the Euro amount of the grant is fixed. What if these assumptions did not hold? What if the grant were some function of the set size?

There are many other interesting directions to explore. We could have more sophisticated utility functions or game designer goals, or we could study “natural” games (i.e. not design mechanisms but look at existing systems). We can study more involved environments; We could study an evolving dynamic environment where new researchers are born and old ones retire. What about composition of multiple systems? Could multiple consortiums form simultaneously or in reaction to other formations? In that case, could there be a domino effect? It will be very interesting to study the relationship between the topology of collaboration networks and consortium composition; our work indicates there may be influence of both connectivity and diameter on the  $SPOA$ . How does the choice of a threshold (which influences the consortium size) influence  $SPOA$ ? Can we propose mechanisms that further improve  $SPOA$ ? The MAGNET game is a multi-round game. There are known results transforming multi-round games to single round but these involve various penalties and assumptions. Can we provide a more efficient reduction in our context? We have assumed the players to be fully rational in their decision making; it will be interesting to study such games in context of bounded rationality and also with players having limited information of their neighborhood as in a distributed network setup.

Finally, we would like to be able to abstractly define, eventually, the class of distributed systems formation games. This will make it easier to understand the various trade-offs and parameters.

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